Circle geometry investigation: Student worksheet

About these activities

In each of these activities you will learn something about circle geometry.

Start by *drawing* the required items in the given circle. Repeat this a few times, either in the same circle or other circles, as appropriate. You will then be asked if you can form a *hypothesis*. You can write simply, perhaps by annotating a diagram.

Then you will *investigate in GeoGebra*, using your diagrams as a basis for what you will create within the software. Using length or angle measurements, as appropriate, you can then see whether your hypothesis is true.

When asked to write a *conclusion*, you should create a diagram that demonstrates what you have found, and write words that explain your theorem clearly. Try to write formally, being specific. Do not just write "the angle". Explain clearly *which* angle you are referring to. After all these are completed, you may be asked to explain one of your conclusions to the class.

The activities proceed more or less in order of difficulty, but do not let yourself get stuck. If you cannot form a hypothesis, you can still investigate in GeoGebra. If you cannot work out how to do something in GeoGebra and timely help is not available, move on to the next activity and come back later.

The most important aspect of all this is the drawing and investigations. Even if you never form a hypothesis or reach a conclusion, you will gain a lot of benefit from the investigations.

Introduction

Use the three circles below to display the following parts of a circle:

* radius
* diameter
* tangent
* chord
* arc
* sector
* major segment
* minor segment.
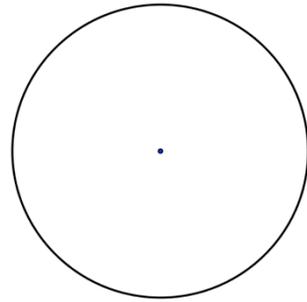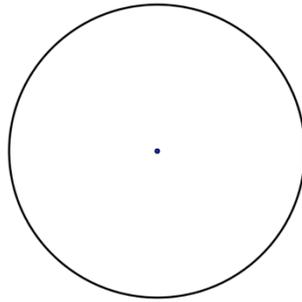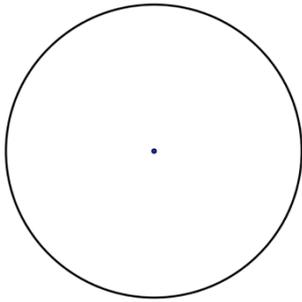
Complete the diagram below.
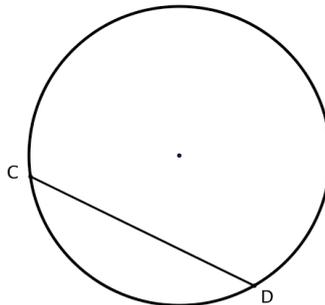
"The chord CD subtends an angle at the circumference of θ°."

Describe this diagram in a similar way.
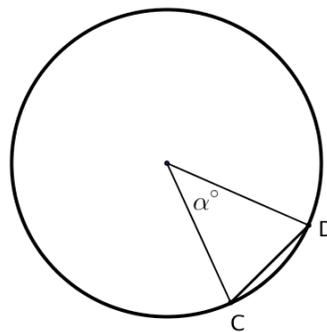
Activity 1

In the first circle, draw a chord.

The chord subtends *three angles* at the circumference (on the same side of the chord); draw them.

Repeat in the other circles with a different chord.
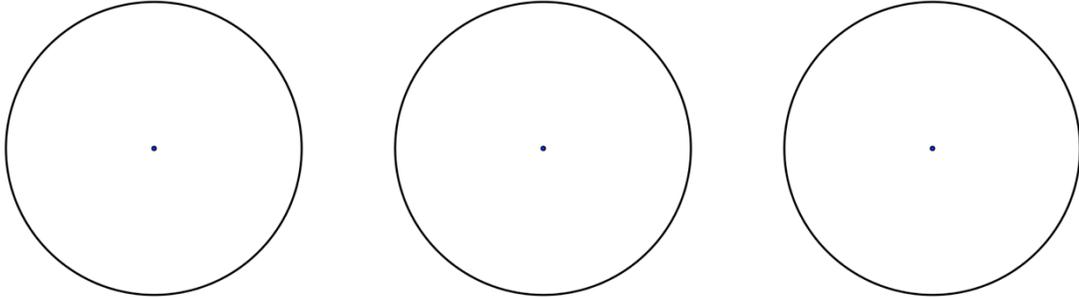
Do you have a hypothesis?

Investigate in *GeoGebra*. Name your file Activity1.ggb.

Conclusion:
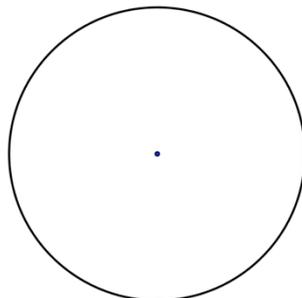
Activity 2

For a given chord, there is a relationship between:
- the angle at the circumference on one side; and
- the angle at the circumference on the other side.

Investigate with different chords in the circles below.
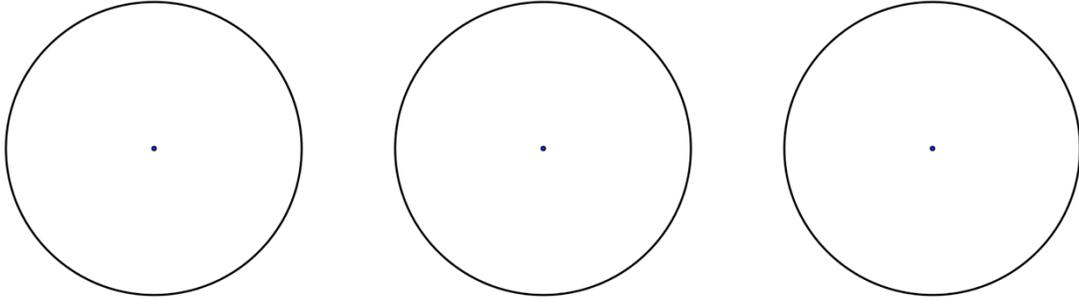
Do you have a hypothesis?

Investigate in GeoGebra. Name your file Activity2.ggb.

Conclusion:
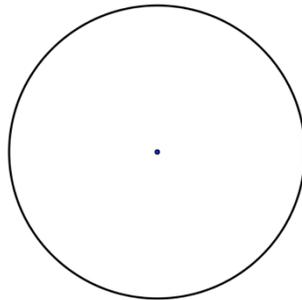
Activity 3

Draw a diameter in each of these circles.

Draw some triangles inside each semicircle (the diameter is the base; a point on the circle completes the triangle.)
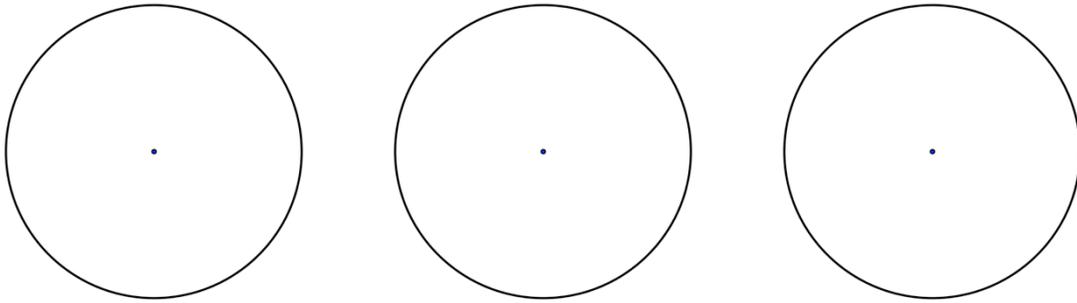
Do you have a hypothesis?

Investigate in GeoGebra. Name your file Activity3.ggb.

Conclusion:
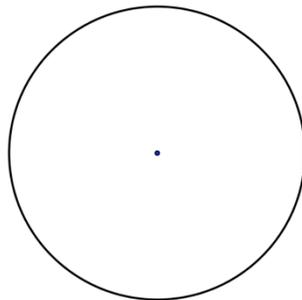
Activity 4

For a given chord, there is a relationship between:
- the angle subtended at the centre; and
- the angle subtended at the circumference.

Investigate in the circles below.
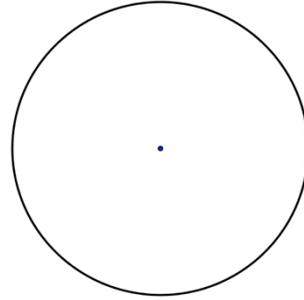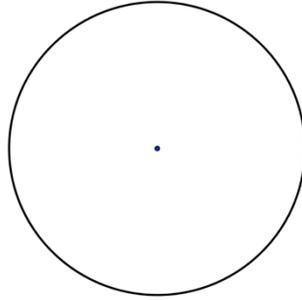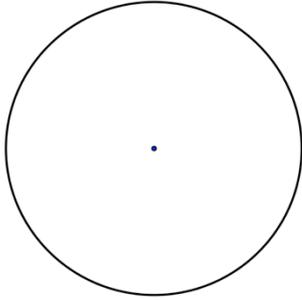
Do you have a hypothesis?

Investigate in GeoGebra. Name your file Activity4.ggb.

Conclusion:
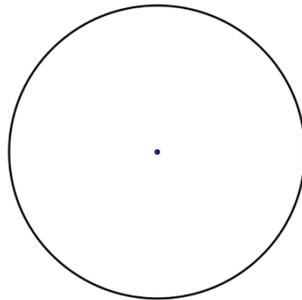
Question 1

Find the values of all pronumerals.

(a)
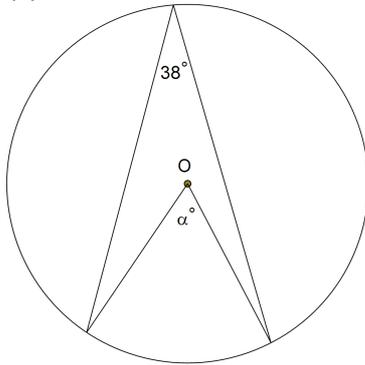
(b)
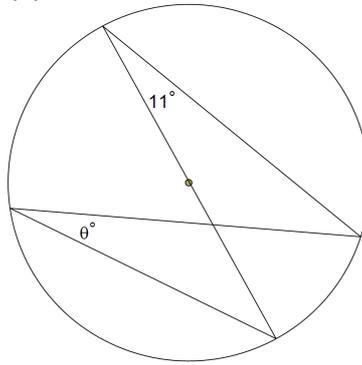
(c)
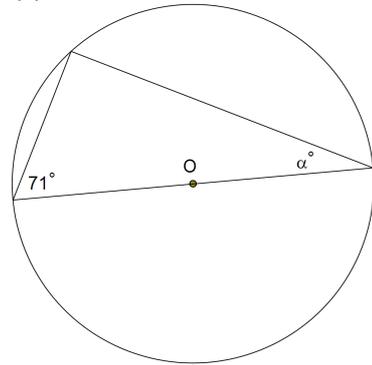
(d)
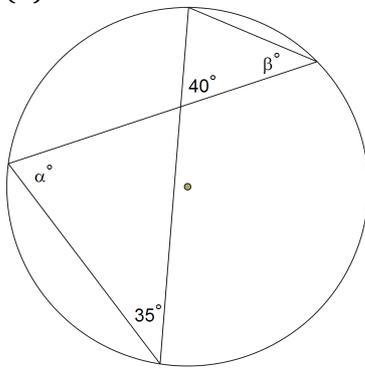
(e)
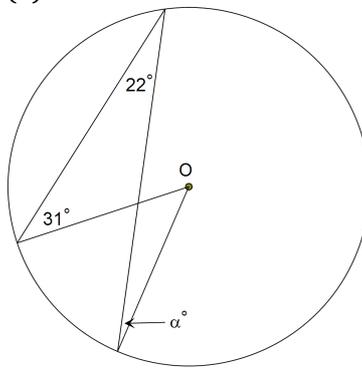
(f)
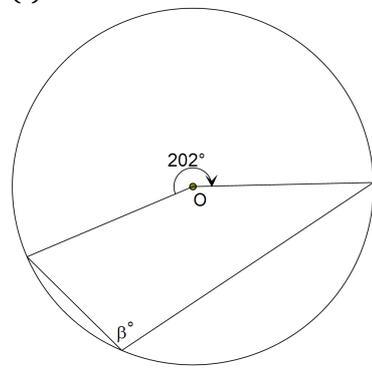
Activity 5

Draw a chord. Join the midpoint of the chord to the centre. Do this a few times.
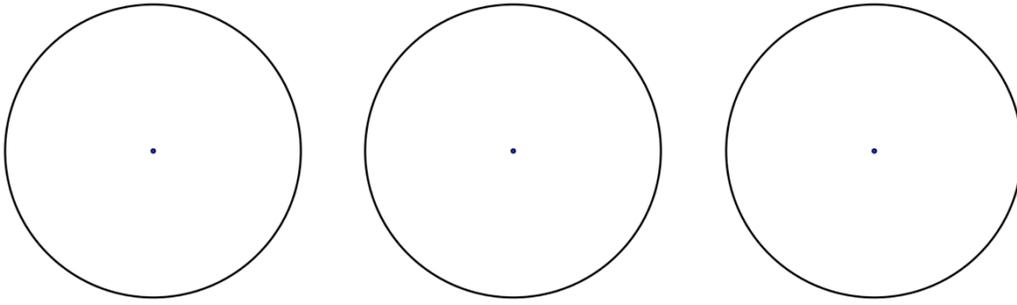
Do you have a hypothesis?

Investigate in GeoGebra. Name your file Activity5.ggb.

Conclusion:
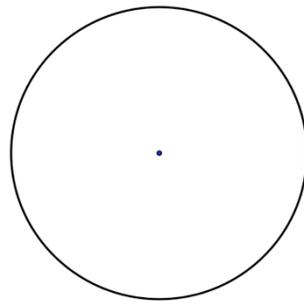
Activity 6

Draw two 2.5 cm chords in the first circle and two 1.5 cm chords in the second circle.
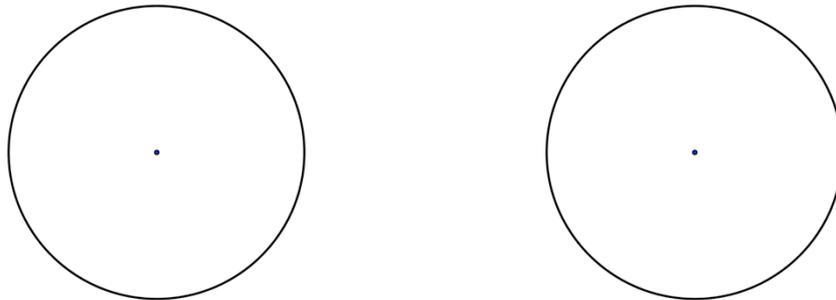
Do you have a hypothesis?

Investigate in GeoGebra. Name your file Activity6.ggb.

Conclusion:
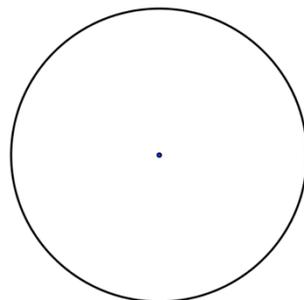
Activity 7

In each circle, draw two chords *AB* and *CD* that intersect at *E*.
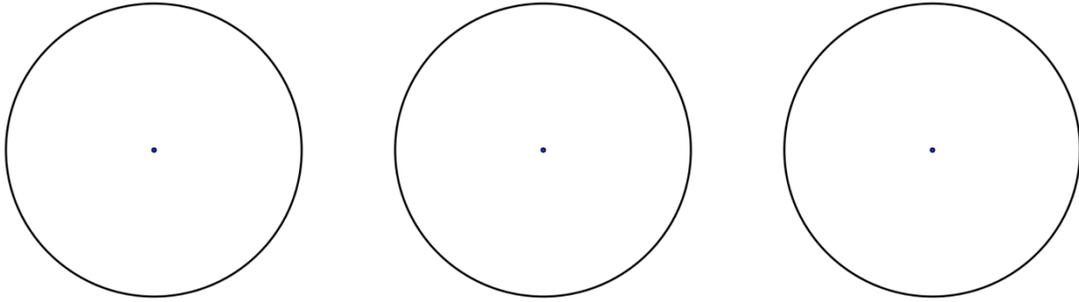
There is a relationship between the four shorter lengths created (called the *intercepts*). It would be very difficult to observe this in a static diagram, so you may skip the hypothesis and…

…investigate in GeoGebra. Name your file Activity7.ggb.

Conclusion:
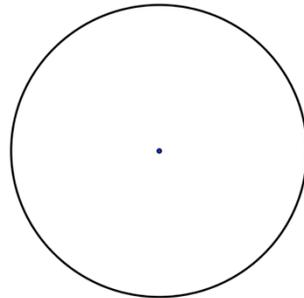
Activity 8

In triangle *ABC* below, draw the perpendicular bisectors of *AB*, *AC* and *BC*.
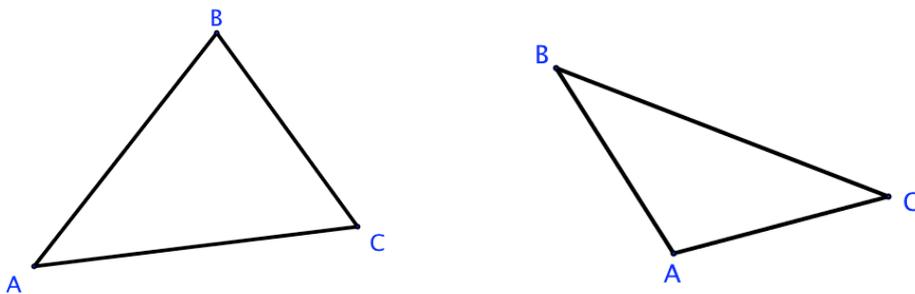
What do you notice?

What is significant about this point? (It is called the *circumcentre* of the triangle.)

Investigate in *GeoGebra*. Name your file Activity8.ggb.

Conclusion:

Activity 9

Draw a *cyclic quadrilateral* in each of the circles below. (A cyclic quadrilateral is a quadrilateral all of whose vertices lie on the same circle.)
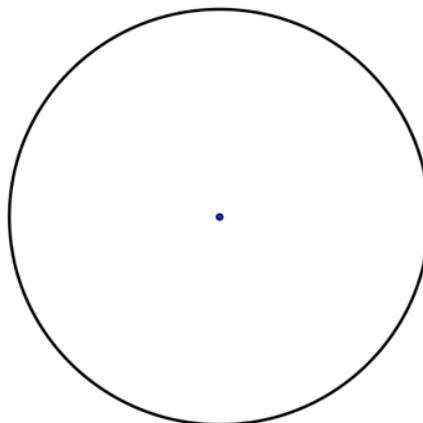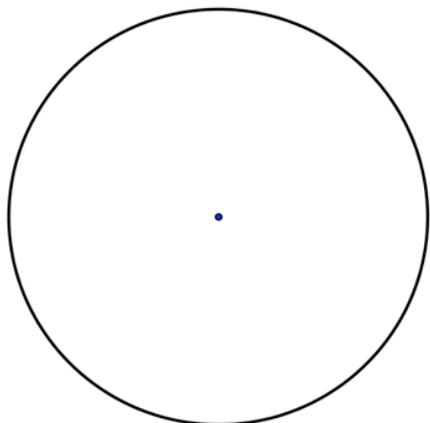
Do you have a hypothesis?

Investigate in GeoGebra. Name your file Activity9.ggb.

Conclusion:
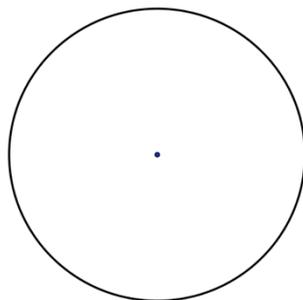
Question 2

Find the values of all pronumerals.

(a)
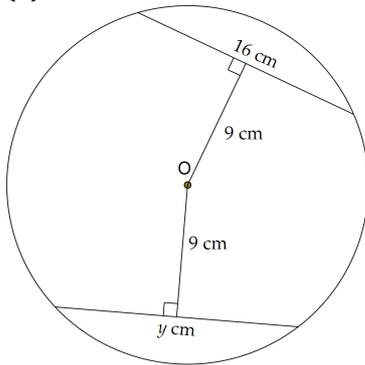
(b)
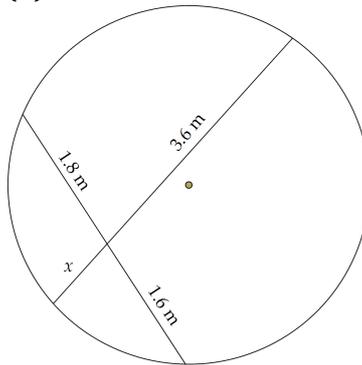
(c)
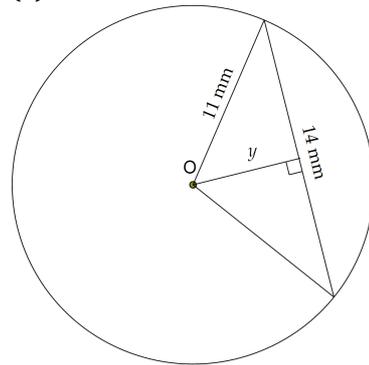
(d)
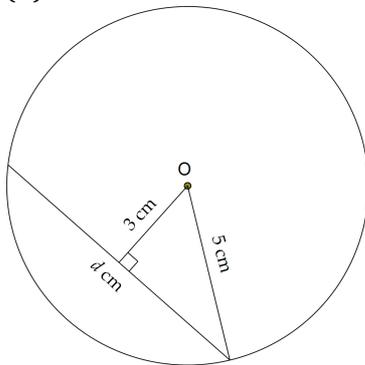
(e)
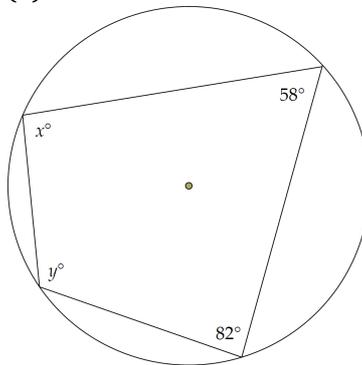
(f)
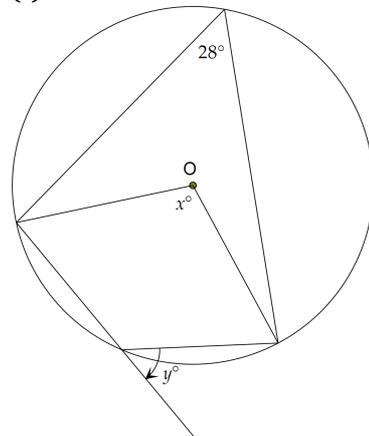
## Activity 10

Draw some radii. For each radius, draw a tangent at the point of contact.
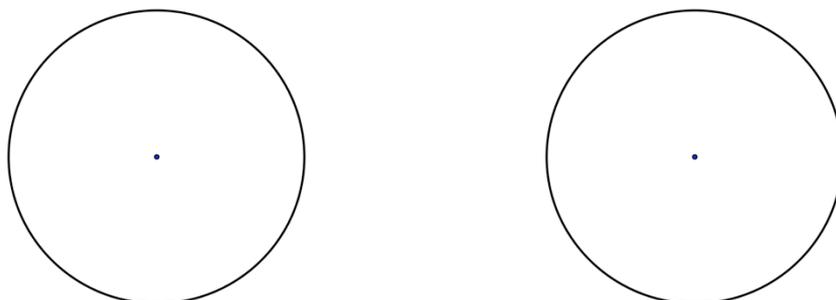
Do you have a hypothesis?

Investigate in GeoGebra. Name your file Activity10.ggb.

Conclusion:
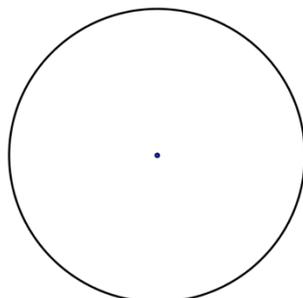
Activity 11

Draw *two tangents* from an *external point* of your choice. Do this a few times.
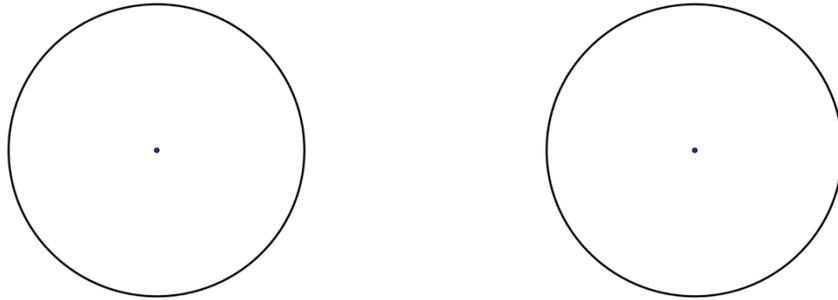
Do you have a hypothesis?

Investigate in GeoGebra. Name your file Activity11.ggb.

Conclusion:
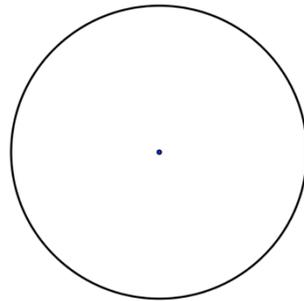
Activity 12

For each circle below:
- draw a tangent
- draw a chord that touches the tangent
- draw a triangle using this chord as the base (of course, the third point must be on the circle).
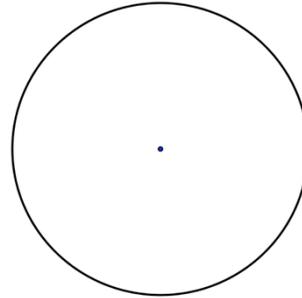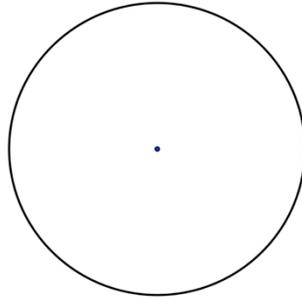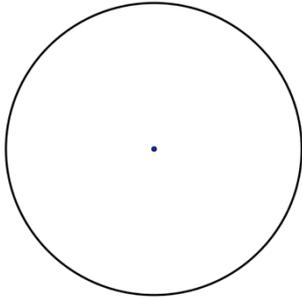
Do you have a hypothesis?

Investigate in GeoGebra. Name your file Activity12.ggb.

Conclusion:
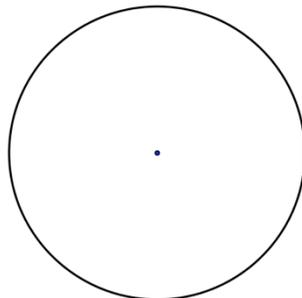
Activity 13

For each circle below:
- draw a tangent (call the point of contact *P*)
- choose a point on the tangent (call it *Q*)
- join *Q* to a point *S* on the far side of the circle; call the other point of intersection *R*.
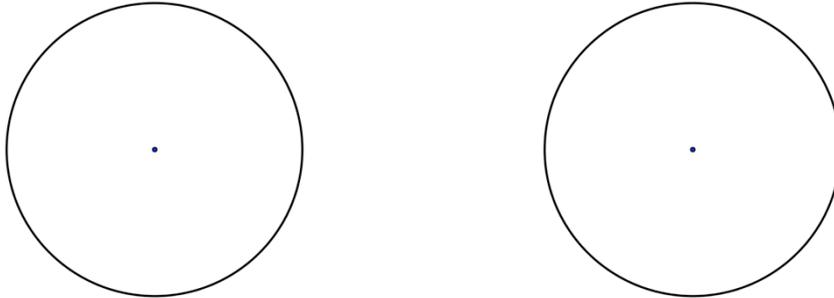
There are several lengths in your diagram. Name them:

Investigate in GeoGebra (Activity13.ggb) and see if you can find a relationship between three of the lengths.

Conclusion:
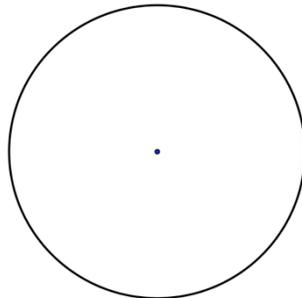
Activity 14

Given a circle and an external point, GeoGebra can create the two tangents from the point to the circle. In this activity, you will *construct* the tangents from first principles.
- Create a circle (centre *O*) and an external point *P*.
- Create a circle whose diameter is *OP*.
- This cannot be done directly; you need to think about it.
- You now have two circles. Place a point at their two points of intersection.
- These are the points of contact for the tangents. Use the ray tool to create the two tangents.
- Move the various points around to confirm that you really have constructed the tangents.

Name your file Activity14.ggb.

Question 3

Find the values of all pronumerals.

(a)
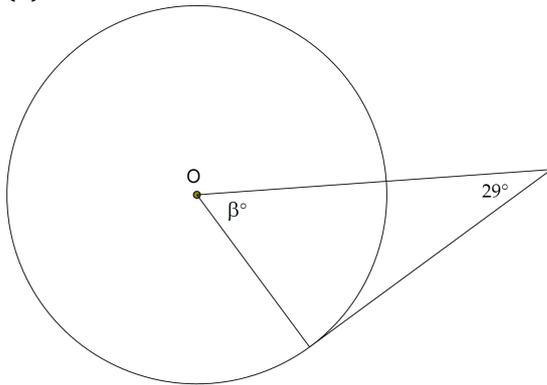
(b)
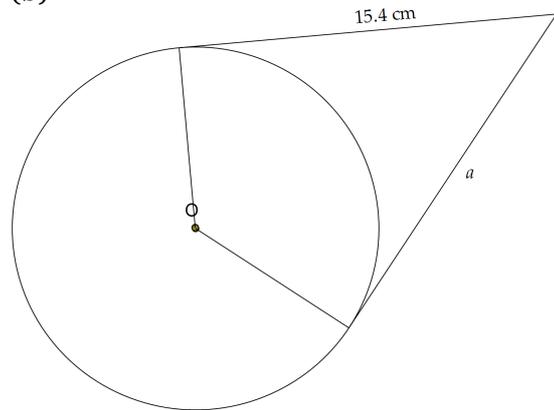
(c)
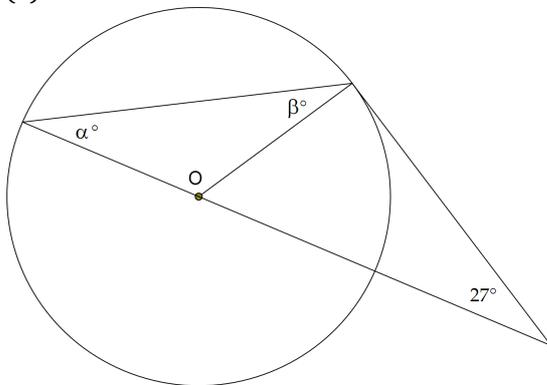
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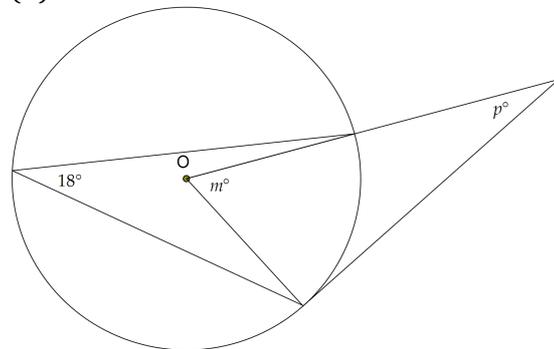
(e)
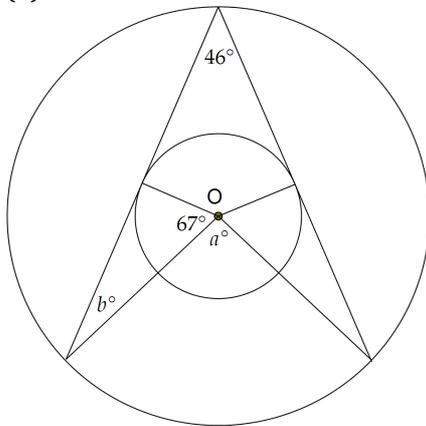
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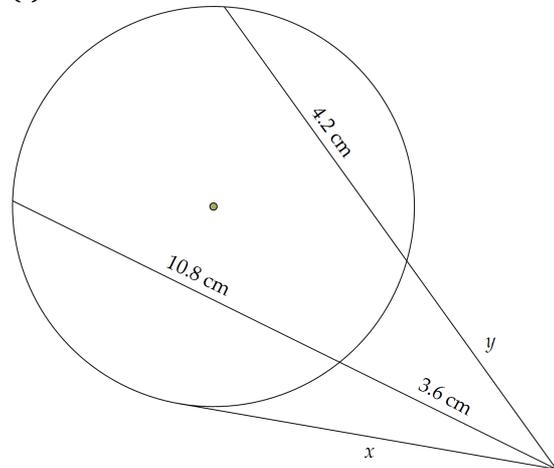
(g)
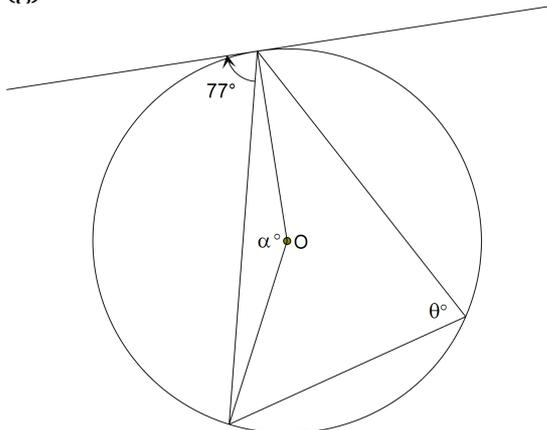
(h) Prove that $\triangle ABC \mid\mid\mid \triangle BDC$.
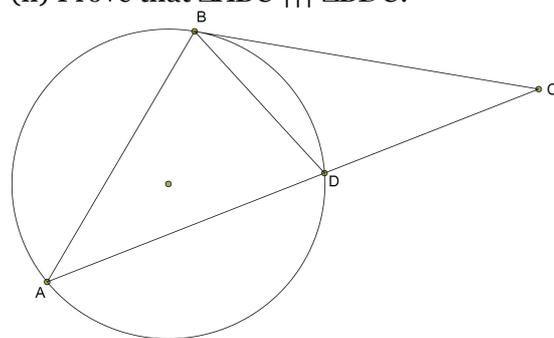
Answers

Question 1
(a) $\alpha = 76$
(b) $\theta = 11$
(c) $\alpha = 19$
(d) $\alpha = 105, \beta = 35$
(e) $\alpha = 9$
(f) $\beta = 101$

Question 2
(a) $y = 16$
(b) $x = 0.8$
(c) $y = 6\sqrt{2}$
(d) $d = 8$
(e) $x = 98, y = 122$
(f) $x = 56, y = 28$

Question 3
(a) $\beta = 61$
(b) $\alpha = 15.4$
(c) $\alpha = 31.5, \beta = 31.5$
(d) $m = 36, p = 54$
(e) $a = 92, b = 23$
(f) $x = 7.2, y = 5.4$
(g) $\theta = 77, \alpha = 154$
(h)   Proof:
      In $\triangle ABC$ and $\triangle BCD$
      1. $\angle C$ is common
      2. $\angle BAC = \angle BCD$ (angle in the alternate segment)
      $\therefore \triangle ABC$ and $\triangle BDC$ (AAA)