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Strategies for Going Mental

Paul Swan and Len Sparrow

Much is known about mental strategies and how children use them. Evidence suggests that discussion should play a key part in the development of mental strategies and yet many mental computation sessions are still characterised by the traditional ten or twenty quick question approach. This paper reviews what is known about mental strategies, examines why a certain level of inertia exists and suggests a way forward.

Introduction

Mental arithmetic is often a focus for debate in the media and across the dining room table. In most cases it refers to the learning and recall of basic number facts and multiplication tables. It is possible however, to use the term mental arithmetic to mean something similar but fundamentally different. Many people would now see mental arithmetic having two parts. The first part is concerned with the recall of facts and the second with the development and use of strategies for calculating mentally. This paper is concerned mainly with the second aspect of mental arithmetic, that of strategy building.

Within the idea of mental strategy building is the issue of whether or not to teach explicitly various strategies or to let them grow and develop as children face and solve problems concerned with mental calculation. This paper adopts the position and assumption that children develop a range of mental strategies by being exposed to rich situations requiring them to explain and describe their method of solution to their peers. In this way they hear and see other strategies to solve problems involving mental computation. This is, however, not an ad hoc, laissez-faire approach as the skilled teacher is aware of the possible variety of strategies and can draw and highlight them in the situation.

The strategies children use to calculate mentally have been researched to the point where we know:

- children invent their own strategies for calculating mentally (Kamii, 1994; Kamii, Lewis & Livingston, 1993);
- children often adopt one method in school and another out of school (Carraher, Carraher & Schliemann, 1985);
- methods vary from child to child and even the same child may choose to use different methods to solve similar problems at different times (Hope & Sherrill, 1987);
- mental strategies differ from written methods: for example, many mental strategies for addition, subtraction and multiplication start from the right,

whereas most mental methods start from the left (Askew, 1997; Hope & Sherrill, 1987);

- the teaching of written methods, particularly at an early age can stifle the development of mental strategies (Carraher & Schliemann, 1985; Kamii & Dominick, 1989);
- some mental strategies are more efficient than others: for example, counting on in ones from a smaller number rather than the larger of two numbers if adding (Hope & Sherrill, 1987);
- strategies have been identified and coded, although strategies are often referred to by different names and codes in the literature (McIntosh, deNardi & Swan, 1996).

Mismatch between what is known and what is taught

Yet despite the increased awareness of how children calculate mentally, many textbooks containing lists of basic fact questions continue to be produced, tables tapes abound and the ten quick mental a day is still practiced in many classrooms. Why is this the case? We would like to suggest there are several reasons for this apparent mismatch between what is known and what is taught related to mental arithmetic.

Tradition

Tradition is very powerful and difficult to change. The rapid-fire, tables drill has been a part of classroom practice since stimulus response theories became popular in the early part of last century. Rigour was valued and drill was viewed as a way of exercising the mind. Parents have come to expect it to be part of mathematics teaching. Principals and teachers value it. Children suffer it. Tradition, by its very nature, is often not questioned. In order to change tradition one must present powerful arguments.

Ease of assessment

Clearly ten quick mental recall questions a day represents a testing rather than a teaching situation. Assessment is clear-cut, results can be monitored, graphed and progress measured. How to measure the development of mental strategies is somewhat more cumbersome.

Defined teaching approach

The rapid-fire approach to mental computation is much more defined and easy to pass on. Simply choose a set of questions which may or may not be related, present them orally, have the children mark the answers and record the result. In ten to fifteen minutes the teacher is able to deliver a neat package.

Discipline

A common practice in schools is to timetable mental arithmetic just before or after a break. It is suggested that mental arithmetic is the ideal activity to settle children to work. The children are controlled, sitting in their seats and have to listen carefully for fear of missing the next question or worse still mixing up the sequence of answers.

Unclear direction

Change will not be effected unless a viable alternative is provided. The old adage 'If it ain't broke why fix it' is often at the core of arguments in favour of keeping the status quo. Viable alternatives that encompass many of the elements that teachers require, while also embracing sound educational principles, need to be offered. The purpose of this paper is to offer a clear alternative to the ten or twenty rapid-fire questions that dominate mental computation sessions in many classrooms around the country.

Developing mental strategies: a way forward

The development of mental strategies is a key element of many Australian and international curriculum documents. The following sample from Western Australia (EDWA, 1998) illustrates the use of the term (Italics added).

> Students choose and use a repertoire of *mental*, paper and calculator *strategies* …
>
> N1.3 uses *counting and other strategies* to mentally solve …
>
> N2.3 … add and subtract one and two-digit numbers drawing mostly *on mental strategies for one digit numbers*
>
> N3.3 adds and subtracts whole numbers and amounts of money and multiplies and divides by one-digit whole numbers, drawing mostly on *mental strategies* for doubling, halving, addding to 100, and additions and subtractions readily derived from basic facts.
>
> N4.3 calculates … drawing mostly on *mental strategies* to add and subtract two-digit numbers and multiplications and divisions related to basic facts

Thompson (1999) described the phrase 'mental strategies' as:

> The application of known or quickly calculated facts in combination with specific properties of the number system to find the solution of a calculation whose answer is not known. They also incorporate the idea that, given a collection of numbers to work with, children will select the strategy that is the most appropriate for the specific numbers involved (p. 2).

It could be argued that children making use of mental strategies are 'working mathematically' and thinking about numbers rather than remembering procedures.

Mental strategies: to teach or not to teach?

A constructivist approach to mental computation relies on the generation and sharing of mental strategies. This places the onus on the teacher to examine and interpret the responses given by children. The teacher therefore needs to have knowledge of mental strategies in order to offer appropriate responses to the children. The response can take many different forms, for example the offering of a question or asking the children for clarification. Armed with this knowledge, teachers then needs to make judgements as to how much advice and help should be offered. Allowing children to discuss and describe the strategies requires teachers to comprehend the nature of the strategy being described. They also need skills to be able to assist children to verbalise their thoughts and communicate them clearly so that the rest of the class can understand.

An alternative approach to developing mental computation strategies from the children's thoughts is to teach a specific strategy in a particular lesson. This teaching approach could be considered almost algorithmic in nature and teachers run the risk of streamlining the use of strategies to the point where flexibility is lost. Flexibility is really the key to the development of skilled mental calculators so it is important to keep this to the fore. It is much easier to teach a specific lesson about a specific mental computation strategy than to work with the possibility of a multitude of strategies. The teacher can focus on a single line of reasoning rather than have to cope with a variety of strategies all at once.

To help with the first approach, which is reliant on the use of student, generated strategies, we suggest the use of routines. The pedagogy of such an approach frees the teacher to focus on discussion rather than on transmitting information in the lesson structure. Teachers and students become familiar with the routine at the same time increasing the amount of time available to focus on strategies that come about due to discussion of methods by the children.

The position suggested in this paper is that rather than teach specific lessons about particular strategies children should explore and discuss a variety of strategies and adopt those that are suited to their needs at that particular time. The following mental mathematics activities have been provided to help teachers who wish to adopt a similar approach to developing mental strategies that relies on explanation and sharing of methods among the class. The aim of any session designed to develop mental strategies should be to develop flexibility in thinking by the children and for them to gain an insight into the structure and properties of number. Askew (1999) suggested that "Tasks that do not 'set ceilings' on the level of difficulty enable pupils to engage with the mathematics at a number of different levels of attainment" (p. 5). Any task given to a class should allow for participation by the whole class and at the same time match the mixed ability nature of the children present.

There are several formats contained in *Think Mathematically* (McIntosh, De Nardi & Swan, 1996) such as *Today's Number Is* and *How Did you Do it?* that encourage children to explore and discuss mental strategies. The *Today's Number Is* activity asks children to list all they know about a particular number. After children become familiar with the format of this type of activity the teacher can encourage children along particular paths.

The *How Did you Do It* activity involves presenting a calculation (29 + 47) to be performed mentally and then asking the children to explain how they went about solving it. Note the use of a horizontal layout of the calculation if it is being shown to the class. This presentation allows for more open responses as children do not immediately equate the calculation with the traditional method.

A variation on the *How Did You do It?* theme used in the following activity.

1. How would you do it? – In your head, on paper or with a calculator.
A question is presented and each child decides the method they would most be inclined to use to solve it. Children are then asked to explain the method and why they chose it.

Another approach involves asking children to list a calculation they would perform in the head, on paper or with a calculator and to explain why they would do it in that particular form.

2. If I know …, then I also know …
Offer children a calculation for example 10 x 5 = 50. Then show that If I know 10 x 5 is 50 then I also know 9 x 5, 11 x 5, 5 x 5, 10 x 50 10 x 0.5 and so on.
Present children with another calculation and ask them to decide what they also know and explain why they know these things (i.e. ask them to explain the connections). Ask them also to show how each calculation is related to the others.

3. I can see.
For example offer the sum 12×18. Then tell the children
I can see $2 \times 6 \times 18$ and also $2 \times 6 \times 9 \times 2$ $4 \times 6 \times 9$ $4 \times 3 \times 2 \times 9$ $4 \times 3 \times 2 \times 3 \times 3$ and so on.
Ask them if some of the calculations above are easier to calculate than the original and to explain their reasons.

4. That's Easy!
Ask children to think of calculation that looks difficult but really is easy to do in the head. Have them explain why the calculation looks difficult but why for them it really is easy.
e.g. $3 \times 2 \times 7 \times 5 \times 5 \times 2$
An example of this activity could be as follows.
That looks hard because there are so many numbers.
Really it is easy because when you multiply it does not matter what order you do the multiplying so the question could be changed to look like this 2×5 (which is 10) multiplied by 2×5 or 10, 10×10 is 100. This only leaves the 3×7 part, which is 21, and this is multiplied by 100 to produce an answer of 2100.

> ***5. Take it Easy***
>
> If you had one wish and could change one number in the following question which one would you change? Explain why.
>
> 17×9
>
> I would change the nine to ten because it makes it much easier to multiply.
>
> How could you use 17×10 to help calculate 17×9?

By developing awareness of the method of calculation and the numbers involved, children are being helped to make sense of calculation. This awareness of calculation, sometimes referred to as *metacomputation*, is an important skill in making a sensible selection of not only calculation method, for example mental, written or calculator, but also calculation strategy.

The value of discussion

Clearly the most important aspect of any activity designed to improve knowledge of mental computation strategies is discussion. Jones (1988) noted that a

> variety of mental methods stimulates conversation and can form the basis of instructive class or group discussion…A child has to think more carefully about his (sic) method to put it into words. Listening to a variety of approaches can inspire him to modify his own methods. The discussion of how methods are linked encourages him to think about the structure of number (p. 43).

Another benefit of discussing mental methods is the development of a mathematical vocabulary. As Jones continued:

> The mathematical vocabulary needed to describe mental methods is extensive …This use of appropriate vocabulary widens a child's conception of the range of situations that may lead to the use of a particular operation (p.43).

Encouraging talk by children about their methods allows a variety of ways to demonstrated. It also keys further discussion about why a particular method was more suitable in this situation, how it works, and what other ways of thinking could be used.

Identified strategies

There are several lists of mental strategies (Rathmell, 1978; McIntosh, De Nardi & Swan, 1996). Some are more detailed than others and some use different terms to describe the same strategy but being able to give a strategy a specific name is not as important as understanding how and why it works. Children often adopt idiosyncratic methods of working, which may blend several different thinking strategies together. Teachers should not expect always to be able to categorise strategies under specific headings. Rather the teaching should focus on explanations by children of how they use the strategies. This can be used to determine the level of understanding they possess. The following list of mental strategies is neither comprehensive nor exhaustive but simply provides an overview of the most common strategies used by children.

Addition and subtraction

Strategy	**Example**
Commutativity	2 + 9. 9 + 2 is easier
Counting on or back	9 + 2, 9, 10, 11
Bridging ten	8 + 5: 8 + 2 = 10, 10 + 3 = 13
Doubles	6 + 6 = 12
Near Doubles	6 + 7 = 12 + 1
Changing subtraction to addition	9 – 7: 7 + ? = 9

Multiplication and division

Commutativity	3 × 8: 8 × 3 = 24
Skip counting	4 × 5: 5, 10, 15, 20
Repeated addition	4 × 5: 5& 5 is 10 & 5 more is 15 and another 5 is 20
Splitting into parts	7 × 4: 5 × 4 makes 20 and double 4 is 8 so the answer is 28
Convert to Multiplication	18 ÷ 3: 3 × 6 is 18
Repeated subtraction	18 ÷ 3: 18, 15, 12, 9, 6, 3, 0
Repeated addition	18 ÷ 3: 3, 6, 9, 12, 15, 18
Counting back	6 ÷ 3: 5, 4, 3 2, 1, 0
Counting on	6 ÷ 3: 1, 2, 3 4, 5, 6

The application of this style of mental mathematics develops children who are confident and competent in such situations. Generally, they have developed:

- a good bank of factual knowledge
- a wide range of mental strategies
- an ability to select from the range for appropriateness
- an ability to articulate their thinking
- an ability to answer quickly

Conclusion

Much of what is suggested in this paper is not new. French (1987) commented about the poor attitude children have toward mathematics and mental mathematics in particular:

> The variety of methods that children and adults use in doing mental calculations is very great and discussion of these in the classroom is very valuable, not to produce a 'best method', but to encourage a flexible approach and make explicit the advantages and insights that come from considering alternatives (p. 39).

He also summed up the key thought when he suggested that the aim of developing mental strategies is to produce flexible thinkers. The ten quick questions approach of mental recall produces panic, fear and anxiety in many children and reduces flexibility of thinking. An approach to teaching mental computation whereby children are taught specific strategies which are practiced may not cause as much anxiety but still may reduce flexibility in thinking as children attempt to apply the teacher's strategy rather than their understood method. Developing mental strategies via discussion should help children gain more flexibility in their approach to solving problems and provide more insight into the properties of the number system. Children will also learn that there is more than one way to arrive at the solution to a problem.

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