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# Connected Understanding

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*This paper was developed from some ideas presented in a plenary paper to the May 1999 conference of the Queensland Association of Mathematics Teachers. It uses reflections on past incidents to discuss the development of connected understanding in maths classrooms.*

## Reflections

I draw here my intimate knowledge of one learner—one of our sons who took interest in maths from a very early age. I watched Peter develop throughout his childhood and questioned him about his growing maths knowledge and skills. He loved playing with and talking about mathematical ideas as a youngster, and still does.

However, he was not an exceptional child. Whenever I tell such stories, many parents and teachers come up later and tell me about similar events, reporting on insights that their children have had. Some contact me after asking their own children the same sorts of questions and being excited by the responses. Many teachers and parents tell stories of children becoming entranced with difficult ideas like infinity, negative numbers and square numbers in their pre-school and early primary years. I find young children's enchantment with mathematics fascinating, and like many grandparents am now delighting in watching the same development in the following generation.

## What's the next number?

We had been travelling for a year, and were north of Mt Isa on the way to "The Gulf". Peter was nearly five, and when the novelty wore off games and songs in the car, we sometimes did number play like "How many wheels will the next truck have?"

One 'game' was "What's the next number?"

*Mum:* What's after fifty six?

*Peter:* Fifty seven. What's after one hundred?

*Dad:* A hundred and one. What's after a billion?

*Peter:* A billion and one. (Pause) What's the biggest number ever?

*Dad:* There is none.

*Mum:* That's right. You can always say another number.

*Peter:* Like one thousand hundred billion million and one?

*Mum:* Yes.

*Peter:* (Long silence, eventually looking teary)

*Mum:* What is wrong?

*Peter:* (Pause) I can't learn to count. I will never be able to count!

*Dad:* Don't worry. You will understand one day.

*Peter:* I do understand. I know it. … It is beautiful.

Perhaps that is the day that Peter fell in love with mathematics—when he realised that it was so great and so abstract that he could not know it.

## Connected levels of understanding

In schools, students are expected to learn about ***objects***–counters, graphs, models, shapes, and later even more abstract objects like variables, theorems and formulae. They are given a rich variety of everyday experiences in coming to understand these objects and to manipulate them appropriately. There are traditions to be followed here, such as grouping, adding, increasing according to a set ratio, rotating, and so on. Thus object-based understandings form a *coherent network of concepts* that about specific objects and the way we operate with them in mathematics.

From these experiences with objects, students develop an understanding of matching ***symbols***. These include symbols that we name and describe objects with, such as the symbols for "two", "limit", or "theta"). They also include symbols used to name and describe actions, such as symbols for "divided by" or "square root of". There are also traditional symbols that represent ideas, such as symbols representing equality and the waves of trigonometric graphs. Thus students reach another level of understanding—a coherent knowledge network of symbols and symbols-based operations. At first the symbols are bound to objects and actions on these, but gradually they become an entity in their own right and thus can be used as foundation objects for further operations and for more complex symbolisation.

The object and symbol levels are somewhat sequential within concepts, but not within time—they apply interactively to the learning of each new concept throughout schooling. Objects are not just concrete manipulatives, and not the province of primary schools. For instance, children might first learning subtraction with base ten materials such as bundles of straws or MAB blocks. To learn factorisation, junior secondary students might also use an object as they draw, partition and rename a rectangle sized x + 3 by x + 2 as $x^2 + 5x + 6$. Senior students might learn about integral notation using drawings of curves and some rectangles that fit under them. Further, any of these students might learn about arrays bank interest through their every day, objective (and also subjective) experience.

Figure 1 (adapted from Schoenfeld, 1986) represents the relationships that I have described to date. The physical and symbolic components each need internal coherence, and direct links need to be made between these two realms.

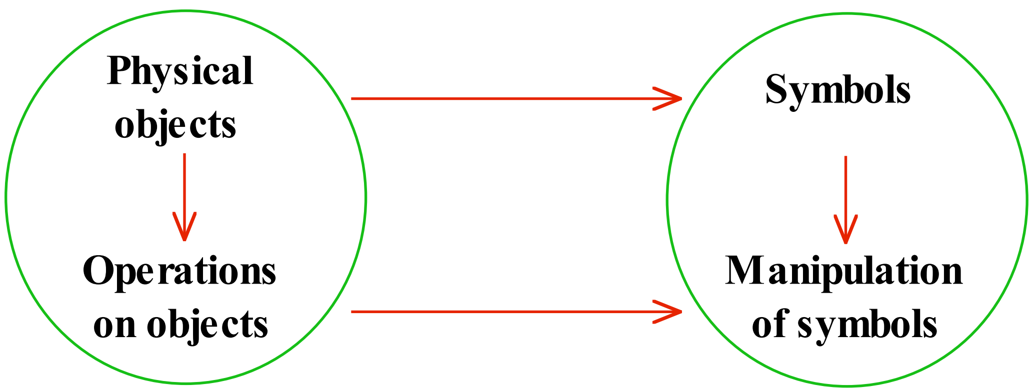


*Figure 1:* Object-based and symbol-based realms of understanding.

A further level of understanding involves ***abstraction*** of symbol-based activity away from objects. Many teachers make good links between concrete experiences and abstract ideas, but don't realise the importance of taking the next step—abstracting mathematics not just out of, but away from, particular objective experiences.

Unfortunately, the symbolic and abstracted realms have received a bad reputation, but this is because they have been used without a strong foundation in the object realm. Developed properly, they are absolutely essential and useful components of mathematical understanding. The challenge here is for students to each be able to recall enough relevant experience for the abstract to remain experientially meaningful as their understanding independent of particular experience[1]. It is in the fact that mathematics is removed from particulars that makes it powerful.

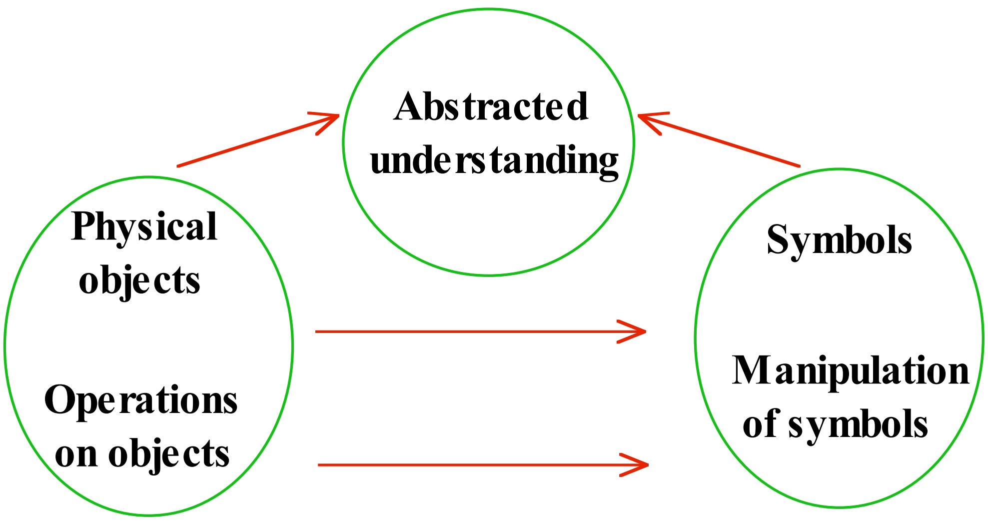


*Figure 2:* Object-based, symbol-based, and abstracted realms of understanding.

It is rare for teachers to encourage students to reach a further level of abstraction—knowledge of *mathematics as an interlocking network of connected ideas*. We do it occasionally, such as when we explain multiplication as repeated addition or decimal fractions to linear measurement, or when linking everyday data with graphs with equations, but generally we do not do it as a matter of course.

Reaching for this level of understanding involves using activities to help students to reflect on inter-relationships between concepts, linking them to create a network of understanding that involves personal meaning and experience.

So instead of teachers being happy that the young children have learnt, for example, that 3 + 6 = 9, they need to push towards more general ideas like:

- one small number of things plus another small number of thing put together make a larger bundle of things
- the bundles can be added in any order (6 + 3 = 9)
- one bundle taken from the sum leaves the other bundle (9 – 3 = 6; 19 – 6 = 3)

[1] Hence I use the term "abstracted" rather than "abstract".

- smaller bundles can be substituted for any bundle (e.g. 3 + 3 + 3 = 9);
- leading to 3 x 3 = 9 and 9 ÷ 3 = 3 (“nine, how many 3s?”)

Most teachers would claim to teach such concepts, but a few probing questions to students show that most children who have learnt the number facts well have never been challenged to generalise at this abstract level. Further, young children who understand these facts and relationships in a generalised, predictive way will also have easy access to 90 – 30 = 60 and 6000 + 3000 = 9000, that 6 tenths plus 3 tenths equals 9 tenths, and that $9a - 6a = 3a$ or $2^3 + 2^6 = 2^9$.

And then comes an even higher level of understanding (see Figure 3) that involves students coming to know about ***themselves as knowers of mathematics***. Again, this not something that only senior students experience—as Peter’s recognition of his own appreciation of the concept of infinity demonstrates:

*Peter:* I do understand. I know it. It is beautiful.

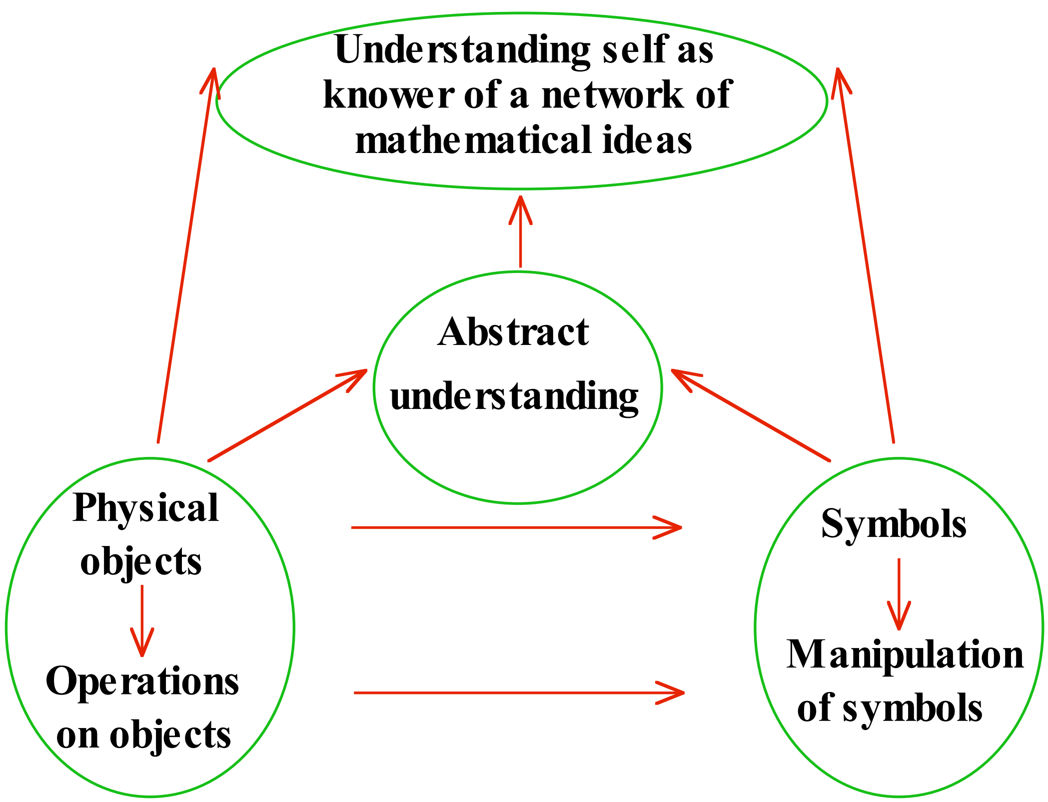


*Figure 3:* Self knowledge: A higher realm of understanding.

## How can we focus on connected knowing?

Teachers allowing and encouraging the development of abstracted, related understandings, as well as the development of knowledge of oneself as a knower of mathematics, involves opening up the curriculum in strategic ways.

One way is to start with children's ideas, and then permit enough freedom in the curriculum to allow the big ideas children have to surface as they explore the objective and symbolic world.

Another strategy for emphasising thinking is to ask more open questions. Consider the following examples (taken from Sullivan & Mousley, 1999) and ask what they have in common.

- The mean height of four people in this room is 155 cm. You are one of those people. Who are the other three?
- A ladder reaches 10 metres up a wall. How long might be the ladder, and what angle might it make with the wall?
- What are some functions that have a turning point at (1,2)?
- A rectangle has a perimeter of 64 cm. What might its area be?

Open-ended tasks engage students in constructive thinking by requiring them to consider the broader possibilities, to seek patterns, to generalise. They are not abstract problems, but they encourage abstraction.

## Conclusion

I am sure that all children are born with the capacity to love mathematics and to see it as a network of changing, meaningful and useful ideas. A lot of the experiences they have in our classrooms do not have these objectives in mind.

I have argued for the use of strategies that allow students to develop four levels of understanding. The first level involves understanding objects and the traditional ways that these are manipulated. The second level, growing out of experience with objects, is understanding of symbols and the ways that these are generally manipulated. The third is the realm or abstracted, connected and increasingly complex concepts. The fourth is a domain of understanding of self as an empowered knower and rational user of mathematics. These levels are not sequential—they develop in parallel.

The result of maths teachers providing experiences that encourage such connected knowing would be many more of Australia's students spontaneously saying: "I do understand. I know it. … It is beautiful."

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